UCRL-ID-133135

Formal Solution for the Fields within a Beam-Bug Calibrator

T.J. Fessenden

July 13, 1998

Lawrence Livermore National Laboratory

Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-ENG-48.

Formal solution for the fields within a beam-bug calibrator

T.J. Fessenden
7/13/98

For some time I was bothered by the fact that measurements of offsets in the various bug calibration setups never agreed with the simple formulae (2) used for determining electron beam position in the Livermore induction linacs and transport systems. About 1983 I realized that the discrepancy arises from the way the bug calibrator simulates an electron beam in a conducting pipe. At that time I solved the problem using the method presented here. Unfortunately, I did not write it up at that time. After considerable effort, I was able to repeat the calculation. Since I have little confidence that after a few years I could ever do it again, I felt obliged to write it up in some detail.

Our beam bug calibrator consists of two conducting cylinders, nominally concentric, that simulate the electron beam within a drift tube. The radii of the larger cylinder is 2.3 times that of the smaller giving an electrical impedance of 50 Ohms to the coaxial combination. To simulate a beam off-axis within a drift tube, the inner tube is moved relative to the outer tube. This only approximately simulates the motion of a beam because the surface current on the inner tube redistributes in response to the translation. Fortunately, the fields of the translated inner cylinder can be found exactly using complex variable theory (1).

Theory

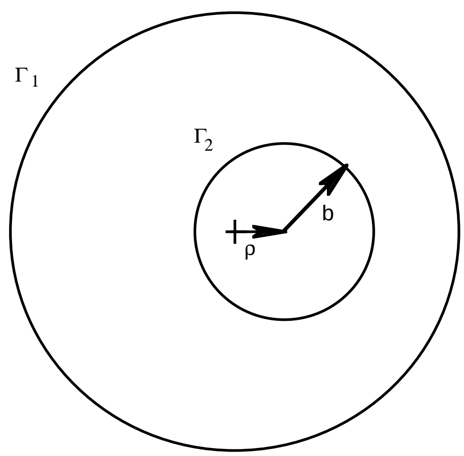

Fig 1. Sketch of a cylinder Γ2 of radius b offset by ρ from the axis of Γ1

Two circular cylinders can be mathematically described in the complex z plane by:

$$\Gamma 1: \ |z| = 1 \text{ and } \Gamma 2: \ |z - \rho| = b.$$

Here z is the complex variable x + iy and ρ is the normalized offset of the axis of the smaller cylinder (Γ2) of normalized radius b. The normalizations are taken to the radius of the larger cylinder. If the coordinates are chosen in the direction of the offset, the parameters b and ρ are real.

Consider the biquadratic mapping

$$(\mathbf{z}-\alpha)/(\mathbf{z}-\beta) = \kappa$$

Here α, β, and k are real. This mapping has the property that circles map to circles or lines (a circle of infinite radius). We define α and β as inverse points of both $\Gamma 1$ and $\Gamma 2$. That is they satisfy the relations

$$\alpha * \beta = 1$$
$$(\alpha-\rho)*(\beta-\rho) = \beta^2$$

Solving for α, β yields:

α[b_,ρ_] := (1 - b^2+ρ^2-Sqrt[(1-b^2+ρ^2)^2-4 ρ^2])/(2 ρ)

βe[b_,ρ_]:=1 / α[b,ρ]

With these definitions the solution for the complex potential between the displaced cylinders as shown in Ref. 1 is given by

$$\Psi = K \ \mathbf{ln}\ (\mathbf{z}-\alpha)/(\mathbf{z}-\beta)$$

The electrical potential is the real part of this complex potential and is given by

$$\vartheta = \mathbf{Re}\Psi = K\ |\mathbf{ln}\ (\mathbf{z}-\alpha)/(\mathbf{z}-\beta)|$$

Note the absolute value signs. The physics of the problem determine the value of K.

The electric field around the circumference of $\Gamma 1$ is given by

$$E = -\partial\vartheta/\partial\rho \ \ \textbf{evaluated at } |\mathbf{z}|=1$$

This is then the surface charge distribution on the inside of $\Gamma 1$ or the surface current distribution in the analogous problem of interest. Evaluating this function taking proper care of the absolute values yields the result for the current density apart from the constant K around the outer cylinder as:

**k[b_,ρ_,ϕ_] := (1 - α[b,ρ] Cos[ϕ]) / (1 + α[b,ρ]^2 - 2 α[b,ρ] Cos[ϕ]) -
(1 - βε[b,ρ] Cos[ϕ]) / (1 + βε[b,ρ]^2 - 2 βε[b,ρ] Cos[ϕ])**

The constant K is found by integrating k around the circumference of the outer cylinder and setting the result equal to the current I. We find

$$K = I/(2\pi\mathbf{r})$$

where r is the radius of the outer cylinder. This constant can also be represented in terms of the voltage Vc generated across the bug with no offset as

$$\mathbf{K = Vc/R}$$

To connect with previous work we note that if the radius of the small cylinder b is set to zero we find $\alpha = \rho$, $\beta = 1/\rho$ and the result presented in Ref (2).

k[ρ,ϕ] = (1-ρ^2)/(1 + ρ^2 - 2 ρ Cos[ϕ])

Defining a position function pos as the voltage difference generated by an offset divided by the centered voltage Vc we find:

pos[b_,r_] := (k[b,ρ,0]-k[b,ρ,Pi])

Performing some algebra yields a simpler form for this position function:

pos[b_,ρ_] := 4α[b,ρ]/(1-α[b,ρ]^2)

Some Results

Let us plot a few of these functions for various cases of interest. First consider the above function that represents a displacement along the line of the pickoffs.

p1 = Plot[pos[.4,ρ], {ρ,0.001,.5}, GridLines->Automatic]

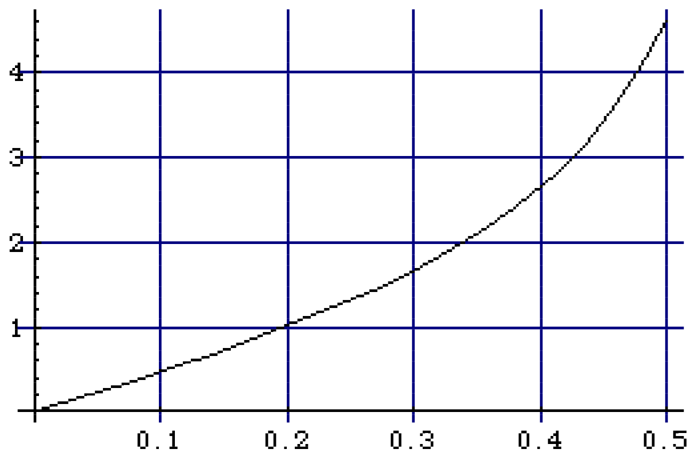

Fig 2. Theoretical plot of the position function versus the normalized radius. The ordinate is (V+ - V-)/Vc where V+ is the voltage toward the offset, V- is the voltage away from the offset, and Vc is the voltage with no offset.

For small offsets ρ this position function is given by :

pos[b_,ρ_] := 4 ρ/(1-b^2)

Although these are mostly of academic interest, it is interesting to look at a number of different cases to fill out the analysis. Let us look at offsets at 30, 45, and 60 degrees as well as at zero degrees to the direction of the pickoffs. For these cases we have a position functions given by

pos30[b_, ρ_] := k[b, ρ, Pi/6]- k[b, ρ, 7 Pi/6]

pos45[b_, ρ_] := k[b, ρ, 0.25 Pi]- k[b, ρ, -0.75 Pi]

pos60[b_, ρ_] := k[b, ρ, Pi/3]- k[b, ρ, 4 Pi/3]

Defining these plots

p2 = Plot[pos30[.4, ρ], {ρ, 0.001, 0.5}]

p3 = Plot[pos45[.4, ρ], {ρ, 0.001, 0.5}]

p4 = Plot[pos60[.4, ρ], {ρ, 0.001, 0.5}]

Finally, plotting these all on the same scale gives:

Show [p1, p2, p3, p4]

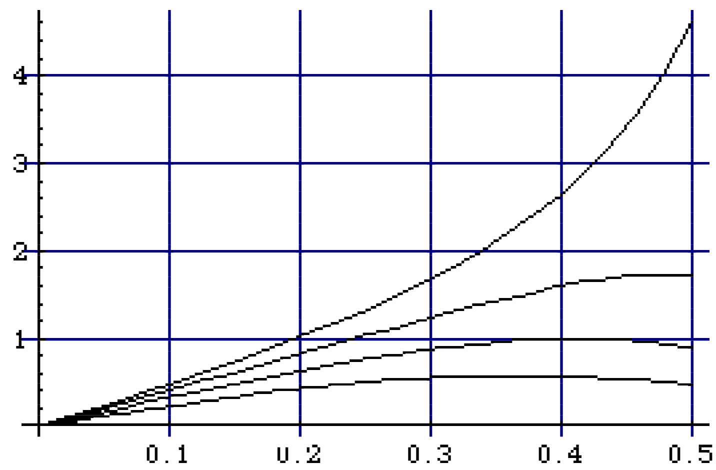

Fig 3. Plots of the position function at angles of 0, 30, 45, and 60 degrees to the direction of the pickoffs for normalized offsets of 0 to 0.5.

As a last case consider the current function formed by pickoffs at the four cardinal points defined as:

cur[b_, ρ_, ϕ_] := (k[b, ρ, ϕ] + k[b, ρ, ϕ+Pi/2] + k[b, ρ, ϕ+Pi] + k[b, ρ, ϕ+3 Pi/2])/4

p5 = Plot[cur[.4, ρ, 0], {ρ, .001, .5}, GridLines->Automatic]

Similarly, at 45 degrees from these points we have

p6 = Plot[cur[.4, ρ, Πι/4], {ρ, .001, .5}, GridLines->Automatic]

Plotting these two curves gives.

Show[p5,p6, PlotRange -> {{0, .5}, {.8, 1.2}}]

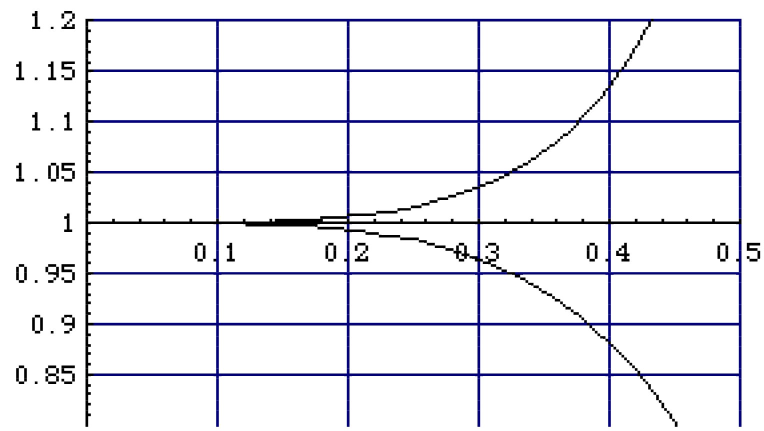

Fig. 4 Plots of the current function for offsets toward one of the pickoffs and half way between pickoffs for the case of a 50 Ohm calibrator.

Comparison with Experiment

John Clark and I performed an experiment to check this theory. A 200 ns pulse was sent down a coaxial line containing a standard beam bug. We displaced the center conductor of the line with respect to the outer conductor and recorded the difference in pick up voltages from each side of the bug as a function of the displacement. The difference voltage was normalized to the on-axis voltage and the displacement was normalized to the radius of the outer conductor. Because of mechanical limitations, the displacement was limited to 15 mm or a normalized displacement of 0.227. These data are plotted below.

```
data = ReadList["data", {Number, Number}];
```

```
ListPlot[data];
```

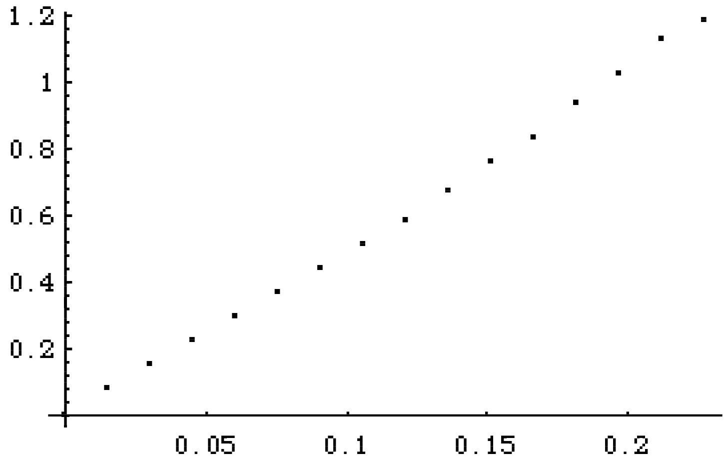

Fig. 5 Measured values of the position function versus normalized radius

For a better comparison Fig. 6 shows the experimental data plotted as points on the theoretical curve contained in Fig 2 above.

```
Show[%, p1, GridLines->Automatic]
```

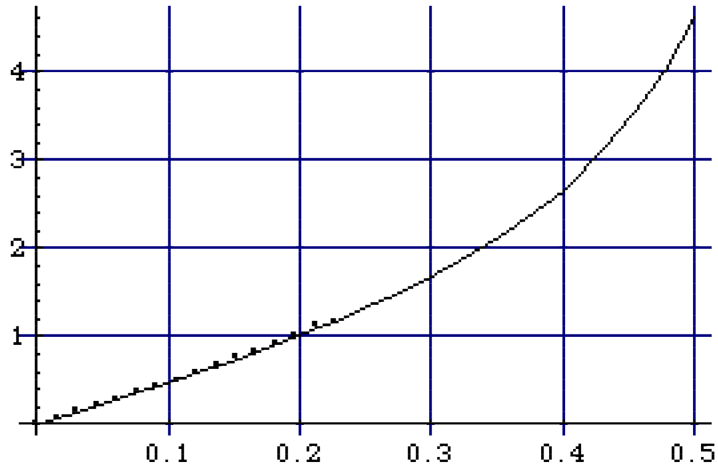

Fig 6. Comparison of the measured values with theory

References

(1) Carrier, Krook, and Pearson, "Functions of a Complex Variable," McGraw-Hill Book Company, (1966), P. 129

(2) T.J. Fessenden, B.W. Stallard, and G.G. Berg, "Beam Current and Position Monitor for the Astron Accelerator", RSI. 43, p. 1789, (1972)